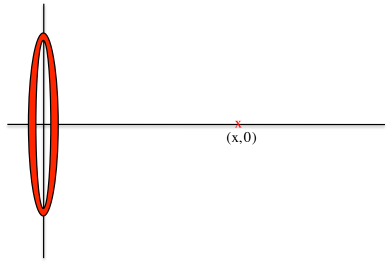
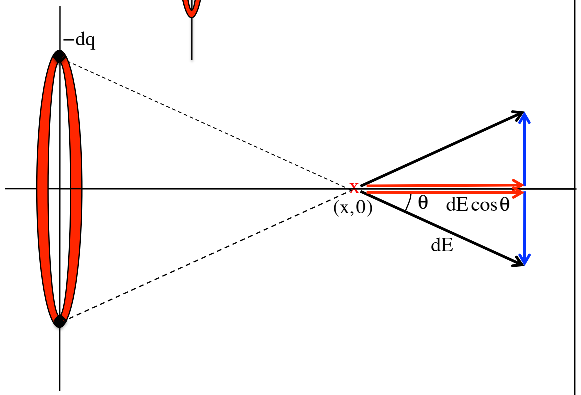
Problem 23.31

A uniformly charged hoop will have an electric field function derives as follows:

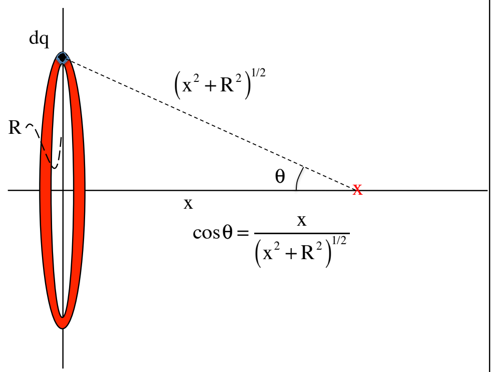
To begin with, notice that the electric field components from differential pieces of charge "dq" on opposite sides of the hoop will have radial (vertical) components cancel one another out (see sketch). That means we only need to determine the net field due to the axial (horizontal) components.





Noting that the cosine function can be defined as shown on the sketch, we can write:

$$|\vec{E}| = \int dE_x$$
$$= \int dE\cos\theta$$
$$= \int \left(k\frac{dq}{(x^2+R^2)}\right)\left(\frac{x}{(x^2+R^2)^{1/2}}\right)$$
$$= k\frac{x}{(x^2+R^2)^{3/2}}\int dq$$
$$= k\frac{x}{(x^2+R^2)^{3/2}}Q$$
$$= (9x10^9\ N\bullet m^2/C^2)\frac{x}{\left[x^2+(.100\ m)^2\right]^{3/2}}(75.0x10^{-6}\ C)$$
$$= (6.75x10^5\ N\bullet m/C)\frac{x}{\left[x^2+(.100\ m)^2\right]^{3/2}}$$



So we have the relationship, using it for the "x" coordinates requested yields:

a.) for r = .0100 m:

$$|\vec{E}| = (6.75x10^5\ N\bullet m/C)\frac{(.0100\ m)}{\left[(.0100\ m)^2+(.100\ m)^2\right]^{3/2}}$$
$$= 6.65x10^6\ N/C$$
$$\text{and } E = (6.65x10^6\ N/C)\hat{i}$$

b.) for r = .0500 m:

$$|\vec{E}| = (6.75x10^5\ N\bullet m/C)\frac{(.0500\ m)}{\left[(.0500\ m)^2+(.100\ m)^2\right]^{3/2}}$$
$$= 2.41x10^7\ N/C$$
$$\text{and } E = (2.41x10^7\ N/C)\hat{i}$$

c.) for r = .300 m:

$$|\vec{E}| = (6.75x10^5\ N\bullet m/C)\frac{(.300\ m)}{\left[(.300\ m)^2+(.100\ m)^2\right]^{3/2}}$$
$$= 6.40x10^6\ N/C$$
$$\text{and } E = (6.40x10^6\ N/C)\hat{i}$$

b.) for r = 1.00 m:

$$|\vec{E}| = (6.75x10^5\ N\bullet m/C)\frac{(1.00\ m)}{\left[(1.00\ m)^2+(.100\ m)^2\right]^{3/2}}$$
$$= 6.65x10^5\ N/C$$
$$\text{and } E = (6.65x10^5\ N/C)\hat{i}$$